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UNSTEADY NATURAL CONVECTION IN HORIZONTAL CHANNELS WITH ARBITRARY WALL TEMPERATURES

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Abstract—Unsteady free convection flow in a horizontal channel with arbitrary wall temperatures has been discussed in general. A physically meaningful exact solution of the problem has been obtained in a closed form by the application of the standard finite sine transform technique. Influences of the governing parameters, the Prandtl number and the Rayleigh number, to bring the flow and heat transfer to final steady states have been discussed separately. For constant values of the arbitrary wall temperatures and of the function, determining the average axial velocity, the final steady state is approached in different times respectively for the cases when the Prandtl number $Pr > 1$ and $Pr < 1$. It is also seen that the function, representing the axial temperature gradient, is influenced by none of the governing parameters; but the steady state flow is influenced only by the Rayleigh number.

NOMENCLATURE

d, distance between the plates;
G, Green's function;
g, acceleration due to gravity;
H, arbitrary function of τ;
k, thermal conductivity;
M, function, defined by equation (20);
m, variable integral number;
N, function, defined by equation (22);
n, variable integral number;
p, pressure;
Pr, Prandtl number;
q, rate of heat transfer per unit area;
Ra, Rayleigh number;
T, temperature;
t, time;
U, nondimensional velocity;
u, velocity;
w, subscript for quantities at walls;
x, x-coordinate;
Y, nondimensional y-coordinate;
y, y-coordinate.

Greek symbols

α, thermal diffusivity of fluid;
β, coefficient of thermal expansion;
ν, kinematic viscosity;
ρ, density;
θ, referring to nondimensional-temperature;
τ, nondimensional time.

1. INTRODUCTION

HEAT-TRANSFER mechanism, involving flows in contact with the walls which undergo a thermal transient change, is important for its applications in various industries. A problem, dealing with transient convection was discussed for the first time by Izumi [1] who considered transient free convection in an infinite circular tube of which the temperature is assumed to be constant along its length. Subsequently Siegel [2] worked on unsteady laminar flow in a duct with unsteady heat addition. The combined transient free and forced convection flow between two parallel vertical plates was discussed by Zeiberg and Müller [3]. Tao [4] has extended it to the case of a flow in a vertical circular tube.

Transient free convection horizontal laminar flow between two infinite nonporous parallel plates has been discussed by only a few workers. Gill and Casal [5] have considered the steady state solution of a particular case of this problem; but the most general case along with a particular case in detail has been considered by Mohanty [6]. He has changed the initial steady state flow into an unsteady one by giving a thermal transient to the walls with the help of a step rise in the plate temperatures; and has considered the flow with constant axial temperature gradient as a particular case. Introduction of step rise into plate temperatures to convert the flow into transient convection is limited in its applications as the case is too ideal for industrial applications. Therefore there arises a need to deal with the problem in a more general way, so that a physically meaningful solution can be found out for the most complicated case. Keeping the physical situations in view, the problem has been reconsidered in the present paper with wall temperatures, given by arbitrary functions. A physically meaningful exact solution of the problem has been obtained in a closed form. It is seen that the nondimensional function, representing axial temperature gradient, is not influenced by the governing parameters, Pr and Ra which are respectively the Prandtl number and the Rayleigh number. If the arbitrary functions, representing wall temperatures, assume constant values, the final steady state is approached in different times separately for the cases when $Pr < 1$ and $Pr > 1$.

2. BASIC EQUATIONS

The equations in Cartesian coordinates for conservation of momenta and energy for a fully developed unsteady incompressible viscous flow between two nonporous horizontal infinite parallel walls are given by

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}, \tag{1}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} = g[1-\beta(T-T_0)] \tag{2}$$

and

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right). \tag{3}$$

Where u, p, ρ, ν, g, T, β and α are respectively the velocity, the pressure, the density, the kinematic viscosity, the acceleration due to gravity, the temperature, the coefficient of thermal expansion and the thermal diffusivity of the fluid. T_0 is the temperature of a reference state. The coordinate axes are chosen such that the x-axis coincides with the lower plate; and the y-axis is perpendicular to it. The upper plate is given by $y = d$. It is to be noted that the viscous dissipation has not been taken into account for considering the balance of energy. Dissipation of energy generally becomes significant in channel flows when the pressure drop is of the order of one hundred atmospheres. Therefore physically it is quite justifiable to neglect dissipation in normal laminar flows where free convection is considered to be of major importance.

Differentiating equation (1) with respect to x, we have $\partial^2 p/\partial x^2 = 0$, i.e. $\partial p/\partial x$ is a function of y and t only.

We write

$$\frac{\partial p}{\partial x} = f(y, t). \tag{4}$$

Hence we have from (2) and (4)

$$T - T_0 = xT_1(y, t) + T_2(y, t). \tag{5}$$

It is to be noted here that for a flow with constant axial pressure gradient, the temperature distribution is given by $T-T_0 = T_2(y,t)$; but we will consider the flow with an arbitrary pressure gradient. Therefore from (3) and (5) we have

$$\frac{\partial T_2}{\partial t} - \frac{\alpha\partial^2 T_2}{\partial y^2} = -uT_1 - x\left(\frac{\partial T_1}{\partial t} - \frac{\alpha\partial^2 T_1}{\partial y^2}\right). \tag{6}$$

Equation (6) is a parabolic equation in T_2. It is possible to obtain a Green's function for $(\partial/\partial t) - (\alpha\partial^2/\partial y^2)$ with the associated boundary and initial conditions. Denoting the Green's function by $G(y, t; y', t')$, the solution of the equation (6) is given by

$$T_2(y, t) = \iint G(y, t; y', t') \times\left[-uT_1 - x\left(\frac{\partial T_1}{\partial t} - \frac{\alpha\partial^2 T_1}{\partial y^2}\right)\right]\mathrm{d}t'\,\mathrm{d}y'. \tag{7}$$

Since T_2 is independent of x, it is desirable that the expression inside the square bracket in (7) should be independent of x. Hence we have

$$\frac{\partial T_1}{\partial t} - \frac{\alpha\partial^2 T_1}{\partial y^2} = 0, \tag{8}$$

so

$$\frac{\partial T_2}{\partial t} - \frac{\alpha\partial^2 T_2}{\partial y^2} = -uT_1. \tag{9}$$

Differentiating (2) with respect to x, and substituting from (4) and (5), we have

$$\frac{\partial f(y,t)}{\partial y} = \rho g\beta T_1(y, t). \tag{10}$$

Considering the nondimensional quantities, defined by

$$\tau = \frac{t}{\mathrm{d}^2},\quad Y = \frac{y}{d},\quad U = \frac{ud}{\alpha},\quad \theta_1 = \frac{\mathrm{d}T_1}{T_0} \text{ and } \theta_2 = \frac{T_2}{T_0},$$

equations (8), (1), (9) and (10) can be rewritten as

$$\frac{\partial^2\theta_1}{\partial Y^2} - \frac{\partial\theta_1}{\partial\tau} = 0, \tag{11}$$

$$P_r\frac{\partial^2 U}{\partial Y^2} - \frac{\partial U}{\partial\tau} = F_1(Y, \tau), \tag{12}$$

$$\frac{\partial^2\theta_2}{\partial Y^2} - \frac{\partial\theta_2}{\partial\tau} - U\theta_1 = 0, \tag{13}$$

and

$$\frac{\partial F_1}{\partial Y} = RaPr\theta_1, \tag{14}$$

where

$$F_1(Y, \tau) = \frac{\mathrm{d}^3}{\rho\alpha^2} f(y, t), \quad Pr = \frac{\nu}{\alpha}$$

is the Prandtl number, and

$$Ra = \frac{T_0 g\beta d^3}{\nu\alpha}$$

is the Rayleigh number.

The set of equations (11)–(14) shows that the flow is completely governed by two parameters Pr and Ra only. It is observed from (12) that when $Pr \to 0$, $\partial U/\partial\tau$ should be quite small in order to have a proper balance of the momentum equation. Physically this means that for a flow with low Prandtl numbers, most of the mass flow takes place under the action of viscous drag. This happens due to the fact that the thermal diffusivity for low Prandtl numbers is comparatively higher which means that the thermal effects penetrate much deeper into the fluid, as a result of which the temperature distribution increases in the central core of the flow. Increase in the temperature distribution retards the fluid flow; because it is observed from equation (2) that increase in temperature increases the pressure gradient along the y-axis which creates hinderance for the longitudinal flow, causing a deceleration for the fluid particles. On the other hand, when $Pr \to \infty$ the process of convection dominates over the effects of diffusion. So the temperature is carried away by the fluid particles. Equation (12) shows that $\partial U/\partial \tau$ should be comparatively larger in this case. At the result of which the horizontal flow is accelerated for comparatively higher Prandtl numbers. It is further observed from (12) and (14) that the Rayleigh number governs the flow, being associated with the axial pressure gradient. Assuming the heating from right (the flow is from left to right), the nondimensional temperature gradient θ_1 is positive. So that the flow is not subjected to the stability considerations. Hence keeping the flow conditions the same, if Ra increases in value, the axial pressure gradient increases. Therefore it is observed from (12) that the flow is retarded for higher values of Ra. It is easy [6] to observe that for a prescribed axial temperature gradient T_1, higher values of Ra create a tendency for instability in the temperature distribution.

3. SOLUTION OF THE EQUATIONS

Initially the fluid has the velocity and temperature distributions, given by

$$\left.\begin{aligned} &U(Y,0) = U_0(Y),\\ &\theta_1(Y,0) = \theta_{10}(Y), \text{ and } \theta_2(Y,0) = \theta_{20}(Y).\end{aligned}\right\} \quad (15)$$

(The values of these functions are given in the Appendix of this paper.) For $\tau > 0$, the wall temperatures, and the pressure gradient start to change with time. These variations may be taken to be arbitrary and simple as follows:

$$\left.\begin{aligned} &\theta_1(0,\tau) = \theta_{1w0}(\tau),\ \theta_2(0,\tau) = \theta_{2w0}(\tau),\\ &\text{and } \theta_1(1,\tau) = \theta_{1w1}(\tau),\ \theta_2(1,\tau) = \theta_{2w1}(\tau).\end{aligned}\right\} \quad (16)$$

The no slip conditions at the walls supply the boundary conditions for the velocity distribution. So

$$U(0,\tau) = U(1,\tau) = 0. \quad (17)$$

The solution of the equation (11) under the imposed initial and boundary conditions is obtained by the application of the standard finite sine transform technique.

It is given by

$$\theta_1 = 2\cdot\sum_{n=1}^{\infty} e^{-n^2\pi^2\tau} \times\left\{ n\pi\int_0^{\tau}[\theta_{1w0}(\tau)-(-1)^n\theta_{1w1}(\tau)]\cdot e^{n^2\pi^2\tau}\cdot d\tau + \int_0^1 \theta_{10}(Y)\sin(n\pi Y)\,dY\right\}\sin(n\pi Y). \quad (18)$$

It is readily noticed from (18) that the axial temperature gradient is governed by none of the parameters Pr and Ra. This is expected physically due to the following fact: as the momentum transfer and heat-transfer properties become effective only in the direction, normal to the solid wall, the physical properties, given by the viscosity and the thermal diffusivity of the fluid, influence the velocity and temperature gradients only in the normal direction. Therefore it is quite normal that the axial temperature gradient should not be affected by Pr and Ra. It is interesting to observe that if θ_{1w0} and θ_{1w1} assume constant values, the final steady state for the axial temperature gradient is approached through oscillations which effectively damp out in a nondimensional time of the order of $1/\pi^2$. Hence in a channel of comparatively greater width, the axial temperature gradient attains its final steady state in a comparatively longer time.

Similarly equations (12) and (14) with the appropriate initial and boundary conditions give the velocity distribution as

$$U = 2\cdot\sum_{n=1}^{\infty} e^{-Prn^2\tau\pi^2} \times\left(\int_0^{\tau}\{-M(\tau)+[(-1)^n-1]H(\tau)\}\,e^{Prn^2\pi^2\cdot\tau}\cdot d\tau + \int_0^1 U_0(Y)\sin(n\pi Y)\,dY\right)\sin(n\pi Y). \quad (19)$$

Where

$$M(\tau) = \int_0^1\Bigg[-2RaPr\sum_{m=1}^{\infty} e^{-m^2\cdot\pi^2\cdot\tau} \times\left\{\int_0^{\tau}[\theta_{1w0}(\tau)-(-1)^m\theta_{1w1}(\tau)] \times e^{m^2\tau\cdot\pi^2}\cdot d\tau + \frac{1}{m\pi}\int_0^1 \theta_{10}(Y)\sin(m\pi Y)\,dY\right\} \times\cos(m\pi Y)\Bigg]\sin(n\pi Y)\,dY. \quad (20)$$

and $H(\tau)$ is an arbitrary function which depends upon the average axial velocity $\bar{U}(\tau)$, given by the continuity equation

$$\int_0^1 U(Y,\tau)\,dY = \bar{U}(\tau);$$

and is presented by

$$F_1(Y,\tau) = RaPr\int\theta_1\,dY + H(\tau).$$

It is seen from (19) and (20) that instead of being influenced by the whole of the temperature distribution at the walls, the velocity distribution is affected only by the axial temperature gradients at the walls. Leaving aside the natures of the functions, $\theta_{1w0}(\tau)$, $\theta_{1w1}(\tau)$ and $H(\tau)$, it is observed that the final steady state for the flow is approached through two modes of oscillations which effectively damp out respectively in nondimensional times of $O(1/n^2\pi^2)$ and $O(1/Prn^2\cdot\pi^2)$. The oscillations, involving dimensionless time $O(1/n^2\cdot\pi^2)$ are introduced only through the axial temperature gradients at the walls. It is observed from (20) that in case of the axial temperature gradients at both the walls are zero both in the initial steady state (see the appendix) and in the state involving unsteady wall temperatures, the final steady state is approached through a single mode of oscillations, involving a dimensionless time $O(1/Prn^2 \cdot \pi^2)$; if $H(\tau)$ is assumed to be constant. Hence for constant values of $\theta_{1w0}(\tau)$, $\theta_{1w1}(\tau)$ and $H(\tau)$, the final steady state is approached in a nondimensional time of the order of $1/\pi^2$, if $Pr > 1$; but if $Pr < 1$ the nondimensional time for the purpose is of the order of $1/Pr \cdot \pi^2$. Hence the flows with higher Prandtl numbers attain the steady state comparatively in a shorter time. As the Prandtl number goes on decreasing, the time for attaining the steady state goes on increasing.

It is further noticed that the Rayleigh number does not affect the time for attaining the final steady state. This is quite natural, as the unsteadiness of the flow is caused mainly due to the temperature propagation which is thoroughly controlled by the response to any thermal change inside the fluid. This response to any thermal change inside the fluid is carried out according to the relative importance of the kinematic viscosity and of the thermal diffusivity, i.e. according to the value of the Prandtl number. Therefore the time for attending the final steady state is controlled only by Pr; but not by Ra, as the Rayleigh number is affected by the joint variation of ν and α instead of depending on their relative importance.

The arbitrary functions $\theta_{1w0}(\tau)$, $\theta_{1w1}(\tau)$ and $H(\tau)$ hinder the flow to attain its steady state quickly. Functions, having larger amplitude, maintain the unsteadiness of the flow for a longer time. If the functions assume constant values, the unsteadiness of the flow is purely due to exponential functions of time.

Solving (13) under the appropriate conditions for θ_2, we have with the help of the same transform technique

$$\theta_2 = 2\sum_{n=1}^{\infty} e^{-n^2\cdot\pi^2\cdot\tau}\left\{n\pi\int_0^{\tau}[\theta_{2w0}(\tau)-(-1)^n\theta_{2w1}(\tau)]\,e^{n^2\cdot\pi^2\cdot\tau}\,d\tau + N(\tau) + \int_0^1 \theta_{20}(Y)\sin(n\pi Y)\,dY\right\}\sin(n\pi Y), \tag{21}$$

where

$$N(\tau) = \frac{1}{\pi}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\left[\frac{1-(-1)^{m+n-p}}{n+m-p}+\frac{1-(-1)^{n-m+p}}{n-m+p}-\frac{1-(-1)^{n+m+p}}{n+m+p}-\frac{1-(-1)^{n-m-p}}{n-m-p}\right]$$
$$\times\int_0^{\tau}\left(\left\{m\pi\int_0^{\tau}[\theta_{1w0}(\tau)-(-1)^m\theta_{1w1}(\tau)]\,e^{m^2\pi^2\tau}\,d\tau+\int_0^1\theta_{10}(Y)\sin(m\pi Y)\,dY\right\}\right.$$
$$\left.\times\left(\int_0^{\tau}\{-M(\tau)+[(-1)^p-1]H(\tau)\}\,e^{Prp^2\pi^2\tau}\cdot d\tau+\int_0^1 U_0(Y)\sin(p\pi Y)\,dY\right)\right)e^{-(m^2-n^2+Prp^2)\pi^2\cdot\tau}\cdot d\tau. \tag{22}$$

The heat transfer is expressed in terms of the Nusselt number defined as $Nu = d(q_{w0}-q_{w1})/kT_0$ where q_{w0} and q_{w1} represent the rates of heat transfer per unit area at the walls $y = 0$, and $y = d$ respectively; and are given by

$$q_{w0} = -k\left.\frac{\partial T}{\partial y}\right]_{y=0}, \quad \text{and} \quad q_{w1} = -k\left.\frac{\partial T}{\partial y}\right]_{y=d},$$

k being the thermal conductivity. Therefore in nondimensional form the Nusselt number is presented by

$$Nu = \left(\frac{\partial\theta_1}{\partial Y}+\frac{\partial\theta_2}{\partial Y}\right)_{Y=1} - \left(\frac{\partial\theta_1}{\partial Y}+\frac{\partial\theta_2}{\partial Y}\right)_{Y=0}. \tag{23}$$

Hence from (18) and (21) we have

$$Nu = -4\pi\cdot\sum_{n=1,3,5\ldots}^{\infty} n\,e^{-n^2\pi^2\tau}\left[n\pi\int_0^{\tau}\{[\theta_{1w0}(\tau)+\theta_{2w0}(\tau)]-(-1)^n[\theta_{1w0}(\tau)+\theta_{2w1}(\tau)]\}\right.$$
$$\left.\times e^{n^2\pi^2\tau}\cdot d\tau + N(\tau)+\int_0^1[\theta_{10}(Y)+\theta_{20}(Y)]\sin(n\pi Y)\,dY\right]. \tag{24}$$

It is observed respectively from (18), (19), (21) and (24) that the axial temperature gradient depends on $\theta_{1w0}(\tau)$ and $\theta_{1w1}(\tau)$, the velocity distribution depends on $\theta_{1w0}(\tau)$, $\theta_{1w1}(\tau)$ and $H(\tau)$, the temperature distribution and the Nusselt number both depend on $\theta_{1w0}(\tau)$, $\theta_{1w1}(\tau)$, $\theta_{2w0}(\tau)$, $\theta_{2w1}(\tau)$ and $H(\tau)$ simultaneously. Taking account of this fact, looking at the forms of the solutions, and considering the manner in which these functions occur in the solutions, it can be concluded that for a flow with given Prandtl and Rayleigh numbers, the axial temperature gradient attains its steady state earlier in comparison with the velocity distribution which is quicker in attaining its steady state in comparison with the temperature distribution. It is noticed from (21) and (24) that unlike θ_2, Nu is influenced by $\theta_{2w0}(\tau)$ and $\theta_{2w1}(\tau)$ through their combinations respectively with $\theta_{1w0}(\tau)$ and $\theta_{1w1}(\tau)$. Hence it is concluded that the unsteadiness of the Nusselt number is maintained for a longer time than that of the temperature distribution. Therefore the rates of heat transfer from the plates become steady only after the velocity and temperature distribution attaining their steady states in the fluid.

It is to be noted from (21), (22) and (24) that although the steady states for the temperature distribution and for the Nusselt number are brought comparatively in a more complicated way through three modes of oscillation, finally it takes the time of the same order of magnitude as is required in the case of the velocity distribution for the effective damping out of these oscillations. Hence for constant values of

θ_{1w0}, θ_{1w1}, θ_{2w0}, and θ_{2w1} and H, the velocity distribution, the temperature distribution, and the Nusselt number attain the final steady states in times which are of the same order of magnitude. Therefore for flows with $Pr < 1$, these quantities require a nondimensional time of the order of $1/P\pi^2$; and for the flows with $Pr > 1$, the time is of the order of $1/\pi^2$ for the purpose. Hence according to the previous discussion, the axial temperature gradient attains the final steady state almost simultaneously along with the other quantities when $Pr > 1$; but when $Pr < 1$ it attains the steady state first; and the other quantities attain it later on.

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APPENDIX

For the initial steady flow, the governing equations are

$$Pr\frac{d^2U_0}{dY^2} = F_1(Y), \tag{a}$$

$$\frac{d^2\theta_{10}}{dY^2} = 0, \tag{b}$$

and

$$\frac{d^2\theta_{20}}{dY^2} - U_0\theta_{10} = 0 \tag{c}$$

where

$$\frac{dF_1}{dY} = RaPr\theta_{10} \tag{d}$$

with the boundary conditions,

$$U_0(0) = U_0(1) = 0, \tag{e}$$

and

$$\left.\begin{aligned}\theta_{10}(0) = \theta_{10w0},\ \theta_{10}(1) = \theta_{10w1}\\ \theta_{20}(0) = \theta_{20w0},\ \theta_{20}(1) = \theta_{20w1}.\end{aligned}\right\}\text{(say)} \tag{f}$$

Therefore we have

$$\theta_{10}(Y) = (\theta_{10w1} - \theta_{10w0})Y + \theta_{10w0} \tag{g}$$

$$\begin{aligned}U_0(Y) = (Ra/12)[0.1(\theta_{10w1} - \theta_{10w0})(5Y^4 - 9Y^2 + 4Y)\\ + \theta_{10w0}(2Y^3 - 3Y^2 + Y)] - 6\bar{U}_0(Y^2 - Y)\end{aligned} \tag{h}$$

where $\bar{U}_0 = \int_0^1 U(Y)\,dY$; and

$$\begin{aligned}\theta_{20}(Y) = {} & Ra[(1/50400)(\theta_{10w1} - \theta_{10w0})^2(50Y^7 - 189Y^5 + 140Y^4 - Y)\\ & + (\theta_{10w0}/1440)(\theta_{10w1} - \theta_{10w0})(10Y^6 - 18Y^5 + Y^4 + 8Y^3 - Y)\\ & + (1/720)\theta_{10w1}^2(6Y^5 - 15Y^4 + 10Y^3 - Y)] - \bar{U}_0[(1/10)(\theta_{10w1} - \theta_{10w0})\\ & \times (3Y^5 - 5Y^4 + 2Y) + \tfrac{1}{2}\cdot\theta_{10w0}(Y^4 - 2Y^3 + Y)] + (\theta_{20w1} - \theta_{20w0})Y + \theta_{20w0}.\end{aligned} \tag{i}$$

It is to be noted that the steady state is governed only by a single parameter Ra, as it is seen from (12) that the flow is governed by Pr only through $\partial U/\partial\tau$. Physically it happens due to the following fact: The action of the viscous drag in a flow becomes effective through the kinematic viscosity. The action of the pressure gradient which is influenced by the axial temperature gradient becomes effective on the flow through the thermal diffusivity. The action of any other force on the flow is revealed through the relative importance of the kinetic viscosity and of the thermal diffusivity i.e. by the Prandtl number. As in the present case the flow takes place only under the action of the viscous drag and of the pressure gradient, it is quite natural that the steady state flow should not be governed by the Prandtl number.

CONVECTION NATURELLE INSTATIONNAIRE DANS LES CANAUX HORIZONTAUX AVEC DES TEMPERATURES DE PAROI ARBITRAIRES

Résumé—On discute sous l'angle général la convection naturelle instationnaire dans un canal horizontal avec des températures arbitraires sur la paroi. On obtient une solution analytique exacte par application de la technique classique de la transformation finie sinus. On discute séparément les influences des paramètres actifs, le nombre de Prandtl et le nombre de Rayleigh, pour amener l'écoulement et le transfert thermique à leur état permanent final. Pour des valeurs constantes des températures pariétales arbitraires et de la fonction déterminant la vitesse moyenne axiale, l'état final permanent est approché pour les cas où $P > 1$ et $P < 1$. On voit que la fonction représentant le gradient axial de température n'est influencée par aucun des paramètres actifs, mais que l'écoulement en régime permanent est influencé seulement par le nombre de Rayleigh.

INSTATIONÄRE FREIE KONVEKTION IN HORIZONTALEN KANÄLEN MIT BELIEBIGEN WANDTEMPERATUREN

Zusammenfassung—Die Strömung bei instationärer freier Konvektion in horizontalen Kanälen mit beliebigen Wandtemperaturen wurde allgemein diskutiert. Eine physikalisch sinnvolle, exakte Lösung des Problems wurde in geschlossener Form erhalten durch die Anwendung der Methode der endlichen Sinustransformation. Einflüsse der Parameter, Prandtl-Zahl und Rayleigh-Zahl, welche den Übergang der Strömung und des Wärmetransports in den endgültigen stationären Zustand bestimmen, wurden getrennt diskutiert. Bei konstanten Werten der beliebig gewählten Wandtemperaturen und der Funktion, welche die mittlere axiale Geschwindigkeit bestimmt, wird der endgültige stationäre Zuständ in unterschiedlichen Zeiten erreicht, je nachdem ob die Prandtl-Zahl $Pr > 1$ oder $Pr < 1$ ist. Es wird auch gezeigt, daß die Funktion, welche den axialen Temperaturgradienten darstellt, von keinem der bestimmenden Parameter beeinflußt wird. Jedoch hängt die Strömung bei stationärem Zustand nur von der Rayleigh-Zahl ab.

НЕСТАЦИОНАРНАЯ ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ В ГОРИЗОНТАЛЬНЫХ КАНАЛАХ С ПРОИЗВОЛЬНОЙ ТЕМПЕРАТУРОЙ СТЕНОК

Аннотация — Рассматривается общий случай нестационарного свободноконвективного течения в горизонтальном канале с произвольной температуой стенок. С помощью стандартного метода конечного синус- преобразования получено физически оправданное точное решение задачи в замкнутом виде. Отдельно рассматривается роль основных параметров, числа Прандтля и числа Релея в достижении потоком и переносом тепла конечного стационарного состояния. При постоянной температуре стенок и постоянном значении функции, определяющей среднюю аксиальную скорость течения, конечное стационарное состояние достигается в различное время для чисел Прандтля $P > 1$ и $P < 1$. Выяснено также, что функция, описывающая аксиальный градиент температуры, не зависит от основных параметров, и только значение числа Релея оказывает влияние на стационарное течение.